1953

Resource Units for High School Mathematics

Mamie Abernathy-McKnight

RESOURCE UNITS FOR HIGH SCHOOL MATHEMATICS

McKNIGHT

1953

RESOURCE UNITS FOR HIGH SCHOOL MATHEMATICS

A Thesis

Presented to

the Faculty of the Department of Mathematics

Prairie View Agricultural and Mechanical College

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

---

by

Mamie Abernathy-McKnight

DEDICATION

It is with sincere devotion
that I dedicate this Thesis to
my Husband

M.A.-M.

ACKNOWLEDGMENTS

The writer wishes to express her appreciation to Mr. Lloyd K. Williams, Thesis Advisor, and Mr. A. W. Randall, Head of the Mathematics Department, for their wholehearted assistance, patience and perseverance throughout the compilation period of this Thesis.

M.A.-M.

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

The increase in activities has created a problem of technique for many teachers. However, some teachers do not understand the many new media. For that reason many systems are providing training for teachers in some form or other.

The Dallas Independent School District Curriculum Study and Improvement Program proposes, wherever the need arises, to develop concrete, practical helps for teachers. The development of resource units as aids to preplanning teaching units is a part of the total plan for curriculum improvement.

During the school year 1951-1952, the first step in this program was initiated with the publication of a manual entitled: The Form For Planning and Developing a Resource Unit. Teachers in the system were expected to use the form for three purposes: (1) for planning and developing a Resource Unit, (2) for preplanning a unit for the classroom and (3) for reporting or recording a unit as planned and developed in class.

After copies of the manual were distributed among the teachers in the system and explanations made thereof, a requirement was set up whereby each teacher was expected to develop and turn in to the Assistant Superintendent in Charge of Instruction at the end of the year, a resource unit.

The writer, a high school mathematics teacher in the Dallas System, met the requirement by submitting a re - source unit entitled: "Solid Figures Come to Life", which is being included in this thesis. Through the development of this unit, a realization of the necessity and usefulness of the resource unit was found, and, through this realization an interest in a thesis composed of several such units was formed.

PURPOSE

The first and foremost obligation of the teacher of mathematics - or, of that matter, any teacher - is to teach effectively. Teaching mathematics in the secondary schools is a task, which if seriously undertaken, will challenge the best efforts of the best teachers. It requires more than a thorough knowledge of the subject matter to be taught, though that, of course, is a must. It requires more, even, than a broad perspective of the field of mathematics in any valid scheme of general education. It demands skill in the techniques of teaching each particular topic or aspect of the subject, in developing generalized concepts, in coordinating generalizations with applications, in discriminating between essential and unimportant matters within the subject, in knowing where to place emphasis and where to anticipate difficulties, in detecting difficulties when they do occur,

in sensing their precise nature, and in knowing how to help the students avoid or overcome them.

The resource unit, if properly developed, can serve as a most effective technique of "getting over to the student" the difficult phases of the subject matter. This fact serves as the main purpose in the writing of this thesis. Furthering the purpose, the writer selected several of the topics in secondary mathematics that have proven through experience to be the most difficult to motivate, thereby causing greater difficulty in teaching.

SCOPE

The Mathematics Department of the Lincoln High School, Dallas, Texas, offers the following courses:

General Mathematics I and II
Algebra I, II, III and IV
Plane Geometry I and II
Solid Geometry
Trigonometry
Business Mathematics

In selecting the units to develop in this thesis, the writer selected at least one topic from each division as listed above (except for Business Mathematics, due to the fact that it is generally accepted as a Commercial subject) in order that an all-around picture may be had of this particular Mathematics Curriculum.

Each unit has been divided into most of the following parts which are defined under the next division of the

Introduction:

I. Title Section
II. Overview - Significance of the Unit
III. Suggestions for Introducing and Motivating the Unit
IV. Teacher Objectives
V. Pupil Objectives
VI. Pupil Problems
VII. Learning Experiences
VIII. Materials
IX. Evaluation
X. Correlation
XI. Teacher Notes and Suggestions for the Unit

These are general headings but in some units it was not necessary to include all of them.

DEFINITIONS

The resource unit or source unit is a carefully prepared series of suggestions for teachers, covering a broad area of work. It contains more material than possibly could be used with any one class. Often it contains much background material for the teacher.[1]

The resource unit is a carefully worked out series of suggestions which the teacher may use in preplanning a teaching unit. There is general agreement that resource units differ from teaching units in that:

---

[1] J. Murray Lee and Dorris May Lee, The Child and His Curriculum (New York: Appleton-Century-Crofts, Inc., 1950), p. 224.

1. They are prepared for use by teachers.

2. They cover a broad area.

3. They contain much more material than is possible to use with any one class.

4. They suggest a variety of possibilities of achieving the same goals.[1]

The resource unit which should be distinguished from the teaching unit of the individual teacher, is merely suggestive in content and arrangement. It constitutes an outline of the broad scope of the unit prepared on the basis of the combined judgment of a group of teachers, supervisors, and the director of instruction in the schools.[2]

Title Section.- Items included in the Title Section are: Title of Unit, Broad Field, Division of Broad Field and Level of Work. All items are self-explanatory, except possibly "Division of Broad Field" and "Level of Work". The term "Division of Broad Field" is used here to indicate a particular phase or division of the broad field; for example, journalism, speech, or dramatics in the broad field of language arts. The term "Level of Work" indicates the

---

[1] Ibid., p. 254

[2] Marie A. Mehl and Hubert H. Mills, Teaching in Elementary Schools (New York: The Ronald Press Company, 1950), p. 120.

placement of the unit; as, for example, fifth grade or second year high school.[1]

The title of the unit should be clear, definite, and concise. It should reflect the center of interest around which the unit develops. It should also present a challenge to the pupil rather than an inference of formal study.

Overview.- The overview should describe the nature of the unit, point out its significance and place in the area for which it is written, and indicate clearly its scope and sequence in the broad field area or subdivision.

Suggestions for Introducing and Motivating the Unit.- Motivation is an important factor in the learning process. Careful planning for introducing the unit is necessary in order to establish readiness for study. Provision may have to be made to interest persons who otherwise would have no concern for study.

Teacher Objectives.- Teacher objectives are of two types, general and specific. It is important for the teacher to distinguish between the types of objectives. General objectives relate the unit to the general aims as stated for the whole curriculum or for the broad fields area. Specific objectives are concerned with the learning situations.

---

[1]Committee on Definitions and Principles, Dallas Independent School District, Manual for Using the Form for Planning and Developing a Resource Unit, 1951, p. 2.

covered by the unit. General objectives relate to long-time objectives. Specific objectives relate to the immediate quickly-achievable objectives of the unit.

Pupil Objectives.- Pupil objectives are specific in nature. They are immediate things the learner wants to accomplish. Teacher objectives and pupil objectives are intimately related, but pupil purposes differ from teacher purposes in intent. The teacher's problem is to examine pupil interests and needs in order to find reasons pupils might have for doing the things that contribute to the realization of teacher objectives.

Pupil Problems.- In setting up objectives for the resource unit, teaching goals are established. Actual planning for learning experiences to contribute to the achievement of objectives begin with the selection of problem situations originating in the objectives and subject-matter scope of the unit. The selection should be made wherein pupils may find worthwhile goals toward which to work. They are planned in light of the teacher's understanding of needs, abilities, and interests existing among class members.

Learning Experiences.- The kinds of learning experiences selected are determined by the types of problems listed. The activities selected reflect the kinds of learning outcomes expected, and while no provision will be made for stating learning outcomes in the units, they will be

kept in mind through continuous reference to objectives. It is the variety of suggested activities that the resource unit may be of most help in planning teaching units. Each objective should have several related activities through which the objective may be realized.

In most cases the problems and activities will be considered in three categories, - initiatory, developmental, and culminating. Initiatory activities or introductory activities are designed to develop readiness for study, to help pupils relate the unit to themselves or to identify themselves with work to be done in developing the unit. The developmental activities constitute the working part of the unit - the content or subject matter - and include various types of activities that pupils engage in to gather and organize information and draw general conclusions. Culminating activities are planned to tie together, to give opportunity to use skills, knowledges, appreciations, and ways of thinking that the unit may develop, and to organize concepts gained in study. Whatever is done in the way of a final culminating activity should leave a sense of completeness to the unit.

Materials.- To assure maximum value in its use for planning teaching units, the resource unit should list materials in detail, including teacher background materials as well as materials for class and pupil use. The subject

matter of the unit is emphasized in listing materials to be used. The list of materials will be of value to teachers in direct proportion to the detail with which they are listed. Listings of reference books, magazines, and other printed materials should be specific, and should include author, title publisher and page number.

Evaluation.- Evaluation is the conscious and discriminating appraisal of teaching and learning for the purpose of improving the processes and the outcomes of each. It is correctly considered as a joint undertaking of teacher and pupils. Evaluation is based upon understood objectives or purposes. It is important for the teacher to know how well each pupil is progressing toward established goals, but it is even more important that the pupil be made aware of ways to evaluate his own progress and that he be caused to have an interest in effecting continuous check on his growth and development.

Correlation.- Correlation refers to the interrelationships among areas of learning. In unit teaching the recognition of this relationship should take the form of natural extensions into other areas.

Teacher Notes and Suggestions.- Under this section any suggestions pointing to use or improvement of the resource unit are used.

CHAPTER II

SAVING AND INVESTMENT - NECESSITIES OF SUCCESSFUL LIVING

Division of Broad Field: General Mathematics

Level of Work: Ninth Grade

Overview - Significance of Unit

The aim of all education must be to prepare the youth of today for their lives of tomorrow - to prepare them to lead happy, useful lives in every possible respect in whatever field of endeavor they may find themselves.

Problems on saving should be given in every grade from the lowest in which formal arithmetic is studied. Questions relating to the income from investments should be introduced as soon as simple interest has been studied. It is not necessary to say to the competent teacher that only such topics should be considered as can be made comprehensible to the student. It is surprising, however, how much of this matter can be placed within the child's reach and made really interesting provided the teacher herself is properly equipped with information, personal enthusiasm, and a belief, grounded in intelligent understanding, in the importance of this subject.

If somehow we could get our boys and young men, and our girls and young women, to develop a definite purpose of saving systematically we should be doing them the greatest personal service.

The main purposes of this unit are to impart important facts that are definite and reliable, direct the pupil's attention to inevitable alternatives, point out that "streaks of luck," or "windfalls" come seldom if ever and, stress the importance of saving and wise investments.

Teacher Objectives

General:

1. To supply an abundance of information to each pupil useful in personal affairs, home and community; e.g., planned spending, the argument for thrift, understanding of necessary dealings with a bank, and keeping an expense account.

2. To mathematically condition pupils for satisfactory adjustment in everyday living.

3. To give pupils a basis for dealing intelligently with the main problems of the consumer; e.g., the cost of borrowing money, insurance, to secure adequate protection against the numerous hazards of life, the wise management of money, and buying with a given income so as to get good values as regards both quantity and quality.

Specific:

1. To teach the considerations that should underlie the investing and saving of money.

2. To teach the importance of a budget.

3. To illustrate the uses and form of a budget.

4. To emphasize saving through the use of a budget.

5. To develop abilities to gather information and set up a budget.

6. To develop an understanding of the significance of cash buying versus installment buying.

7. To emphasize the use of negotiable papers.

8. To develop a thorough understanding of the values to be obtained through thrift and investment.

9. To emphasize the principles of investment.

10. To develop an understanding of the meaning of stocks and bonds and their usefulness.

11. To develop a realization of the need of carrying life insurance, hospital insurance, fire insurance, auto insurance and others.

12. To develop an understanding of some of the principles upon which insurance is based.

13. To emphasize the carrying of these principles of saving and investment into their homes and communities.

Pupil Objectives

1. To learn the necessity of keeping a budget.

2. To learn the significance of cash buying versus installment buying.

3. To become familiar with negotiable papers.

4. To learn the value of thrift and investment.

5. To acquire an understanding of the purpose and usefulness of insurance.

6. To enjoy gathering data for own budget.

7. To collect ideas, facts, objects of high interest through interviews with agents.

8. To enjoy contributions of others.

9. To learn about specialists working in investment agencies.

Pupil Problems and Needs Anticipated in This Unit

To learn in detail about:

1. Budget
2. Cash Buying versus Installment Buying
3. Negotiable Papers (Forms furnished students)
4. Thrift and Investment
5. Insurance

Learning Experiences - Activities

I. Budget

A. Each pupil is taught to keep a budget of his own expenditures on forms provided by the teacher.

1. Necessity of planning before spending emphasized
2. Check made weekly to determine if expenditures were in accord with the weekly estimates

B. Each pupil is to make an itemized list of all his expenditures for the year
   1. Clothing
   2. Recreation and amusement
   3. Etc.

C. Each pupil collects data showing cost to heat, light and rent a house.
   1. Each pupil brings data concerning his own home
   2. Information obtained from contractors and realtors
   3. Average cost for lighting obtained from Dallas Power and Light Company

D. Data from government statistics showing cost of living standard.
   1. Classified according to the divisions of budget
   2. Make problems in figuring percentage in making classifications

E. The making of the budget
   1. Its usefulness
   2. How it helps to save money
   3. Savings and thrift emphasized
   4. Making of budget thoroughly explained

F. Model budgets collected and studied

G. Each pupil makes a budget for a year for a typical family - man, wife, boy and girl (each in high school) - with an income of $295 per month.

H. Each pupil compares own expenditures with the boy or girl of the budget

1. Discussion as to whether or not they are demanding an unfair portion of their family's income

I. Ways of cutting down expenses investigated

1. Problems showing saving when buying in large quantities
2. Problems showing saving when buying for cash
3. Problems showing savings when buying "best quality" goods

J. Ways of having greatest enjoyment at least expense.

II. Cash Buying versus Installment Buying

A. Problems in finding the true interest rate when buying on the partial payment plan

B. Problems from many sources secured by pupils showing interest rate actually being paid in Dallas

C. Ways of building up credit at bank studied

D. Problems showing saving made by borrowing at bank and paying cash

E. Salary and personal loan companies investigated showing true rate paid at these companies

III. Negotiable Papers

A. Proper way to write a check

B. Bank drafts, time drafts, trade acceptance, notes

C. Endorsements

D. Bank discount

IV. Thrift and Investment

A. The necessity of saving while earning

B. How a savings bank operates

C. How money invested regularly grows

D. How to build up an income

E. How to invest an accumulation of savings

F. Loaning money on mortgage

G. Investing in bonds

1. Price and yield

2. Judging the safety of a bond

3. Stability of bonds

H. Principles of investment

1. Safety of principal

2. Certainty of income

3. Rate of return

4. Marketability

5. Diversification

I. Preferred stock as an investment

J. Investing in common stock

K. Diversified investments

V. Insurance

A. The need of carrying life insurance

1. Protection to dependents

2. Provision for old age
3. Protection against disability

B. Average estates left by men at various ages
1. Showing made of safe way to save
2. Insurance as a certain way to save, premium must be paid or lose insurance

C. Different kinds of life insurance
1. Ordinary
2. Twenty payment
3. Endowment

D. Kind and amount of insurance to carry
1. Graphs made showing cost increase with age
2. Cash surrender value
3. Dividends
4. Relation of salary, dependents, insurance

E. Fire insurance
1. Dallas fire insurance zone map
2. How to determine amount to be carried
3. Determining if own home is properly insured

F. Auto Insurance
1. Personal liability
2. Property Damage
3. Fire and Theft

Materials

1. Statistical government reports

2. Budget forms

3. Collection of light, gas and water bills

4. Model budgets

5. Bank interest rate sheets

6. Material on various loan companies showing the interest rates

7. Sample bank drafts, time drafts, trade acceptance, notes

8. Stock exchange data

9. Materials from various insurance companies

10. Beatley, Ralph, "Arithmetic for a Free Society," Mathematics Teacher. 40:276-278, 1947.

11. Buckingham, James, "The Social Value of Arithmetic," Twenty-Ninth Yearbook of the National Society for the Study of Education. Part I (Bloomington, Illinois: Public School Publishing Company, 1917. p. 38.

12. Dunn, Allen, Golthwaite, and Potter: Useful Mathematics. New York: The Macmillan Company, 1950.

13. Grossnickle, F. E., "Concepts in Social Arithmetic for the Eighth Grade Level," Journal of Educational Research, 30:475-488, 1937.

14. Hogben, Lancelot, Mathematics for the Million. New York: W. W. Horton & Company, 1937.

15. Lennes, N. F., The Teaching of Arithmetic. New York: The Macmillan Company, 1923.

16. Tripp, M. O., "The Teaching of Financial Mathematics," School Science and Mathematics, 44:425-531, 1944.

17. Woodring, Maxie N. and Sanford, Vera, Enriched Teaching of Mathematics in the Junior and Senior High School (revised edition). Bureau of Publications, Teachers College, Columbia University, New York, 1938. 133pp.

Evaluative Procedures

I. The Doe Project

The Does own their six room home in Dallas, Texas, valued at $8,000. It is heated by a floor furnace, an electric range is used for cooking. The home is mortgaged for $3,500. It is a straight mortgage bearing interest at 6% payable semi-annually. The size of the lot is 50' x 102'. New paving assessment of $176 is to fall due June 10. All of the assessments are paid. The furniture is all paid for. They own a 1949 Chevrolet. Mr. Doe carries two life insurance policies: A 20 year policy for $5,000, taken out when he was 22; an ordinary life policy for $3,000, taken out when he was 26.

The Doe family consists of Mr. Doe, 38, now employed as bookkeeper at a salary of $3,420 per year, Mrs. Harriot Doe, age 36, Henry, age 15, sophomore in high school, and Elsie, age 16, junior in high school.

Using the outline given below, make a budget for the Doe family.

Your grade on this work will depend on the reasonableness and the work-ability of your plan by the detail to which you go in making the budget.

BUDGET FORM

1. Savings

   a. Life Insurance

b. Other forms of savings

2. Shelter

a. Rent, or
b. Home ownership
   1). Principal payment
   2). Interest
   3). Taxes
   4). Insurance
   5). Upkeep

3. Food
a. Groceries (Miscellaneous)
b. Meat
c. Vegetables
d. Milk
e. Bread
f. Lunches

4. Clothing

a. Mr. Doe
   1).
   2).
   3).
b. Mrs. Doe
   1).
   2).
   3).
c. Henry (boys work out in detail)
d. Elsie (girls work out in detail)

5. Operation

a. Heat (kind)
b. Light
c. Water
d. Range (gas, coal, or electric or combination)
e. Household supplies
f. Telephone
g. Repairs
h. Service
   1). Laundry
   2). Barber
   3).
i. Insurance on furniture and household goods

6. Advancement

a. Church
b. Benevolence (community fund)
c. Health insurance
d. Dentist
e. Oculist
f. Doctor
g. Books, magazines and papers
h. Clubs
i. Lodges
j. Henry's allowance
k. Elsie's allowance
l. Mrs. Doe's allowance
m. Mr. Doe's allowance
n. Car
    1). Payment per month
    2). Depreciation
    3). Insurance
        a). Fire and Theft
        b). Property Damages
        c). Personal Liability
    4). Gas
    5). Oil
    6). Tires
    7). Repairs

The expenditures in each subdivision should just total to equal the allowance for that budget division. The total of the six budget divisions should equal the income for the year.

II. A project for the student's own home, modeled on the Doe's project was formed.

III. The question: "Am I taking a fair share of my family income?", will be asked after finishing the two projects.

Correlation

Commercial Subjects - A knowledge of saving and investment from a mathematical standpoint will serve as a great

aid in helping students understand the same topics in the Commercial Department.

History - In studying the banks and insurance companies the student develops a method for doing historical research.

Home Economics - Under Clothing in the budget the student gathers information concerning the kinds of clothing, costs and durability of goods.

Economics - Practically all items in the budget will be studied in some form in the Economics course offered in the Social Studies Department.

Generally, correlation can be made with most of the divisions of study because of the enormity of terms the student becomes familiar with in the various research studies throughout the unit.

CHAPTER III

ALGEBRA AND ITS LANGUAGE

Division of Broad Field: First Course Algebra

Level of Work: Ninth Grade

Overview - Significance of the Unit

It is the purpose of this unit to provide resources for the teaching of First Year Algebra in the secondary schools with the view toward vitalizing the mathematics program. Too often Algebra is taught as an end within itself rather than the means to the end for which it was intended. Algebra is a language of symbolism which forms the basis for the understanding of the wonders which have been wrought in science and industry with the aid of mathematics.

It is felt that the concept of Algebra as a language should receive greater and more vital emphasis; that more of our boys and girls might grasp its meaning and open doors through its use that lead to further progress and success. The student should be led to discover the amount of algebraic skill needed for definite work in physics and other sciences, to discover application of graphs to phases of statistics and business problems, and to discover equations, formulas, and number relationships in other fields.

A foreword and note to the teacher in using the following material might well be found in the following three criteria:

1. The Logical Criterion which has reference to the structure and organization of mathematics as a science.

2. The Social Criterion which indicates a concern with the usefulness of arithmetic in life's affairs.

3. The Psychological Criterion which is concerned with the total child and how he learns.

(Resourceful teachers and students of the curriculum rightly contend that it should always be possible to organize a program for learning in which the innate structure of mathematics harmonizes undamaged with social situations which take full account of the ways in which children can best learn.)

Suggestions for Introducing and Motivating the Unit

1. Have a short "get acquainted" talk about the history of mathematics in general and Algebra in particular. (Mathematics the basis of our technical age and Algebra the basis of higher mathematics.)

2. Open a discussion of possible course objectives for the purpose of starting students to searching, and verbalizing about their uses of, and objectives in mathematics.

3. Undertake visits to local industry to get an on the scene picture of mathematics in action.

4. Indicate the scope of Algebra as a language emphasizing the need to become familiar with its symbols and terminology.

5. Emphasize the fact that man's power to earn a living and enjoy life is dependent to a great degree on his ability to think in quantitative terms, to recognize logical relationships, and to use the power of generalization.

6. Point out the importance of science in our civilization, and that mathematics is the language with which science talks and thinks.

7. A discussion introducing the varied uses of Algebra, stressing that mathematicians use the same tools, pencil and paper, used by high school students in their mathematics classes, are busy solving vital problems of the day, aiding engineers discover new oil fields, improve telephones, manufacture better and more efficient machinery, design better airplanes, etc. Lead the students into an understanding of the necessity of grasping fundamentals to better acquire the experience to accomplish these feats. Work toward building their eagerness, that they wish to see for themselves, Algebra's problem solving power.

Teacher Objectives

General:

1. To aid pupils to grow in their ability and desire in acquiring the meaningful concepts necessary for quantitative thinking.

2. To point out the importance of Algebra as a cor-

relating factor in the whole mathematical set up.

3. To aid the pupil in realizing the power of symbolism, not only in mathematics but in every phase of human endeavor.

4. To emphasize through history that the development of algebraic knowledge has influenced development in science.

5. To point out the uses of algebra and its phases in science, industry, business administration.

Specific:

1. To emphasize the language concept of algebra in both the symbolism and vocabulary.

2. To determine the present numerical ability of each student, with the idea of beginning study where the pupils are.

3. To emphasize the different types of reading ability necessary in algebra and other mathematics; illustrating the concise language of algebra.

4. To emphasize and illustrate the importance of the four fundamental operations in algebra.

5. To illustrate the nature of algebraic numbers, emphasizing the interesting background of our number system.

6. To illustrate in every case where possible the practical application of algebraic processes.

7. To illustrate the similarity between algebraic fractions and arithmetic fractions.

Pupil Objectives

General:

1. To understand the purpose and part which mathematics has to play in our lives.

2. To develop the ability to read and investigate and make considered judgments and decisions not only in mathematics but in all phases of life.

3. To discover the part played by mathematics in the construction of the airplane, radio, automobile, and other technological developments.

4. To relate mathematics to other areas of study, classifying and understanding the concepts found in physics, chemistry, business, social studies, music, machine construction, etc.

5. To gather practical problems by interview, visitation, and research reading, which are found in the fields of science, business, social science, construction, etc., for use in the classroom.

6. To gain confidence in one's own ability by a coordinated plan for study and action.

Specific:

1. To understand that algebra is a language of sym-

bolism which has a history in its development b y man.

2. To understand that algebra is generalized arithmetic and that its symbolism has to be learned through study and practice.

3. To understand that in algebra letters represent numbers.

4. To understand the meaning and use of the equation and its parts.

5. To build a mathematical vocabulary and know the meaning of such terms as:

a. like terms, b. unlike terms, c. numerical coefficient, d. polygon, e. regular polygon, f. quadrilateral, g. parallelogram, h. constant, i. equation, j. formula, k. horizontal, l. literal number, m. parentheses, n. quadrant, o. subject of a formula, p. variable, q. vertical, r. function, s. factoring, t. integer, u. prime numbers and many more.

6. To appreciate the order and concise nature of mathematics.

7. To be able to use directed numbers in addition, subtraction, multiplication, and division of monomials and simple polynomials.

8. To know the important rules of algebra just as one knows the rules of football or baseball for only in that manner can one succeed in either.

9. To understand the relationship between a formu-

la and an equation and to understand how formulas are derived experimentally.

10. To understand the nature of graphs and their use in algebra.

11. To understand the nature of exponents and radicals and their use in algebra.

Pupil Problems and Needs Anticipated in This Unit

1. The language and symbolism of algebra.

2. The nature of algebraic numbers.

3. Special products and factoring.

4. Fractions

5. Functional relationships involved in fractional equations.

6. Functional relationships involved in linear systems.

7. Functional relationships involved in quadratic equations.

8. Exponents and radicals

Learning Experiences - Activities

1. Show that algebra is not unlike other fields of endeavor in the use of symbols by collecting a list of symbols or abbreviations from newspapers, radio, business, and government affairs.

2. Make reports on history of numerical and literal

notation.

3. Make a table of scores of the World Series or Dixie Series to introduce the idea of two related numbers changing together by results taken from a game or sport in which all are interested.

4. Plot the graph of the World Series or the Dixie Series and show by the graph that as one variable changes, the other does also.

5. Make a list of known rules; change them to formulas; solve certain ones of them and graph them.

6. Translate word statements into algebraic language.

7. Get a weather report in which hourly changes in temperature are given as so many points plus or minus.

8. Plot q quarter of a football game showing yards gained or lost, emphasizing more this principle of the directed number.

9. Make drawings illustrating that the perimeter of a rectangle is a function of the width.

10. Obtain from clerks their methods of subtracting.

11. Locate points on a map illustrating the similarity in principle of plotting points on a graph.

12. Plot the route of two cars, knowing rate and time of starting, having them leave two located towns and meeting each other.

13. Illustrate meaning and derivation of a formula

by having class observe similarities and dissimilarities between centigrade and fahrenheit thermometers; lead them to derivation of formula.

14. Illustrate the algebraic relationships between numbers and the plotting of those relationships.

15. Have a class report on the history of the changes in radical signs.

16. Have a class report about the life and work of Pythagoras and other men in the field of mathematics.

17. Utilize various resource people at appropriate points during the school year.

Materials

1. Materials gathered from various industries employing mathematicians.

2. Materials gathered from papers, magazines, etc.

3. Various mathematical textbooks.

4. Materials gathered from other areas such as physics, chemistry, civil service, business, radio, etc.

5. Visual aids.

6. Algebra, a correlating factor in the field of mathematics:

a. Mathematics Teacher, XXII:2, Feb/, 1929. "Graph as a Part of Cooperative Mathematics."

b. School Science and Mathematics, XXXI: 5, May, 1931. "Trigonometry in High School";

XXI: 9, Dec., 1931. "Cooperative Mathematics Helps Introduce the Formula."

c. Making a Living, L. S. Lyons

d. School Science and Mathematics, XXXI: 3, March, 1931, "Physics as a Career."

e. Handbook on Applied Mathematics

f. Seventh Yearbook of National Council of Teachers of Mathematics, pp. 126-133.

g. School Science and Mathematics, XXXIII, April, 1933, p. 426, "A Mathematical Room That Speaks for Itself"

7. Algebraic Concepts used in Science, Business, Construction, etc.

a. Jansson, Harper, and Agnew. Handbook of Applied Mathematics, Van Nostrand, 1936.

b. Mathematics Teacher, XXV, Oct., 1933, "Mathematical Abilities in Physics, pp. 313-331.

8. Algebra as a factor in the development of scientific research:

a. Marshall, L. C., The Story of Human Progress, Macmillan, 1931, pp. 150-167.

b. School Science and Mathematics, XXXV:13, March, 1935, "The Romance of Research"; XXXV Jan., 1935, "Algebra as a Medium for Interpretation and Control of Nature", pp. 6 - 17.

Suggested Evaluative Procedures

1. Ask students to prepare an outline of the material covered.

2. Have students make up a test covering the material. Tests are to be discussed, evaluated and con-

clusions reached as to what is a good test.

3. Institute a general discussion of the material covered to uncover false notions, encouraging questions and leading a general review.

4. Devote a period to solving miscellaneous problems from all areas covered.

5. Prepare test over material covered and administer test.

6. Discuss test and take steps to clear up deficiences.

7. Check evaluations on group work, reports, given, pupil interest.

8. Try to decide which areas of teaching are weakest and which parts of unit were best received.

9. Hand out questionnaires to students to ascertain their opinion of course, method of teaching, their own standing, etc.

10. Evaluation should be a continous process which helps the student become increasingly aware of the need for self-evaluation.

Correlation

1. Make a list and study the algebraic language used in departments of science, business administration, art, drawing, and home economics.

2. Seek problem content in everyday mathematics: temperature variation business gains and losses, deposits and withdrawals, ball games won and lost, etc.

3. Arithmetic: Correlate problems worked in arithmetic and algebra; understand the contributions that algebra has made to arithmetic.

4. English: The wording of problems and spelling of new words.

5. Social Science: The language used in compiling statistics, mortality tables, banking problems, problems of finance in government, transportation, daily living, etc.

6. Art: Ratio and proportion - posters.

7. Industrial education: formulas, use of geometric figures, etc.

8. Speech: "Number Stories of Long Ago", history of mathematics, etc.

9. Latin: Words as monomial, binomial, etc. Illustrate their derivation from Latin and the corresponding English word.

Suggested Teaching Units.

1. Introduction to Algebra
2. Paving the Way for Algebra
3. Algebraic ideas and processes
4. Formulas

5. Equations
6. Positive and negative numbers
7. Graphs
8. Systems of linear equations
9. Operations with polynomials
10. Products and factoring
11. Algebraic fractions
12. Powers, roots and radicals
13. Quadratic equations
14. Ratio, proportion and variation
15. History of Algebra and its place in the mathematical world.

CHAPTER IV

THE MEANING AND USE OF GRAPHS

Division of Broad Field: Second Year Algebra

Level of Work: Tenth Grade

Overview - Significance of the Unit

In this modern world of business and industry, we are almost daily confronted with quantities whose values change from time to time. Perhaps one of the most useful functions of mathematics is to develop a method of showing the changes in units by means of pictures called graphs.

There are five important types of graphs: the bar graph, broken line graph, curved line graph, circle graph and the picture graph. Each of these graphs is specially adapted to particular data, therefore, it is important that we select with care the type of graph to be used in each case.

Statements of related facts, when expressed in words, are difficult to understand, but when these facts are compared by means of graphs you can readily see at a glance just what the words did not express at first. Graphs then are used to express related facts.

a. Circle graphs are used for budget making.

b. Line graphs, straight, broken or curved graphs - magnitude and importance of events for various periods of time.

c. Bar graphs are used for unrelated comparisons.

Suggestions for Introducing and Motivating the Unit

Employ graphs to motivate the drill on fundamental mathematical concepts and processes.

The concept of relationship between numbers (How one number may vary with another) is one of the most fundamental and important in mathematics. It serves to integrate the course.

Teacher Objectives

General:

1. To develop a deeper understanding and appreciation of the importance of formulas in our modern world.

2. To develop the ability to read and analyze written problems.

3. To encourage the habit of careful observation and thinking.

4. To develop a desire to want to do and be specific in problem solving.

5. To develop skill in securing accurate data and real facts that are valuable for problem solving.

Specific:

1. To develop a knowledge of graphs in their relation to everyday life.

2. To develop an understanding and social aspect in the use of graphs.

3. To develop skill in interpreting and making graphs.

4. To develop skill to solve practical problems by the use of graphic techniques.

5. To learn to read graphs, to understand their meaning and to use them in solving problems. Graphs are so commonly used that they may be considered almost a part of our language.

6. To aid pupils in acquiring benefits and enjoyments in the interpretation of graphical illustrations.

7. To develop skill in locating points on axes.

8. To be able to determine a common solution of a pair of linear equations.

9. To develop skill in solving equations having two unknowns.

Pupil Objectives

1. To introduce the facts of relationship by means of a life situation.

2. To determine how to use formulas as a means of expressing relationships.

3. To point out what graphs to use for particular data.

4. To make all measurements accurate.

5. To determine that graphs not only express relationships but may actually serve as the solution of problems.

6. To label graphs with the proper titles.

7. To develop skill in drawing graphs and using them in solving problems.

8. To make formulas for linear equations in which the graph passes through the origin.

9. To determine the common point of solution on a graph.

Pupil Problems and Needs Anticipated in This Unit

1. To show relationships expressed in a rule.

2. Time table for roasting beef. - If a three pound roast of beef is to be well done, it should be cooked one and one-half hours. How long should four pounds of roast be cooked? Five pounds of roast? Ten pounds of roast?

3. What relationship exists between the numbers showing weight and the required time of roasting the meat well done? Formulas: $h = 1\frac{1}{2}\ p$; $h = \frac{1}{2}\ p$ rare.

4. Draw a line graph showing time for roasting beef well done and rare.

5. Draw graphs showing the relationships in the following formulas. $P = rt$, $C = np$.

6. The highest temperatures in Dallas for the first ten days in January are: January 1 - 40° F; January 2 - 40°; January 3 - 42°; January 4 - 44°; January 5 - 33°; January 6 - 36°; January 7 - 31°; January 8 - 33°; January 9 - 40° and January 10 - 38°. Draw a line graph showing

this information.

Learning Experiences - Activities

1. A brief review of bar and picture graphs.

2. Secure a bulletin board displaying graphs from newspapers and magazines.

3. Strive for accuracy and elegance in work.

   a. Class committee posts the work of the members of the class on the bulletin board in order of excellence.

   b. The standard to be observed in marking bar graphs, line graphs, circle graphs, etc., should develop out of discussion on graphic materials posted.

4. Employ graphs to motivate drill on fundamental mathematical concepts and processes. Topics that give much practice are ratio, common ffactions, decimal fractions, percentage, standards of measurement, measuring angles with a protractor, use of the metric scale, directed numbers and formulas.

5. Employ a carefully planned technique. The pupil should take one step at a time to do his work with a clear understanding. Example: To draw a graph for a simple formula; 1. Understanding the principle, 2. Tabulating the related pair of numbers to determine the graph to be

made; 3. Draw the axes; 4. Choose the scale and select the unit; 5. Locate the points; 6. Draw the graph; 7. Check the graph, and 8. Interpret the graph.

6. Using graphs to teach the idea of relationship.

Materials

1. Textbook (First Course in Algebra by Mallory), pp. 115 - 160.

2. Graph blackboard, compass, colored crayon and ruler.

3. Daily newspapers, magazines, Texas Almanac, Agriculture Yearbook, Weekly Readers, Family Budget

4. Attendance Office Records
   a. Roll book showing the average daily attendance for a class for six weeks.
   b. Cumulative Record Cards
   c. Report Cards (class grades)
5. Health Statistics - Record of weight and growth
6. Production records
   a. Agriculture
   b. Manufacturing
   c. Natural resources
7. Temperature Charts
   a. Fahrenheit thermometer
   b. Rainfall records

8. Athletic contests

a. Baseball

b. Football

c. Track

d. Basketball

9. Sanford, Vera, Short History of Mathematics. New York: Houghton Mifflin Co., 1930.

10. Reginal, J., Practical Mathematics. New York: Stevens Kimball National Education Alliance, 1942.

Suggested Evaluative Procedures

1. Decide whether graphs are essential in this modern age.

2. More practice in drawing graphs and using them in the solution of problems.

3. Construct graphs of formulas.

4. Pupils compare tables and formulas and see the correspondence between them.

5. Devote a period to construction of the four types of graphs and a method of expressing their relationship.

6. Introduce formulas as a third means of expressing relationships.

7. Teaching to do independent thinking by having oral discussions.

8. Represent equations graphically as an important

aid in reflective thinking.

9. Practice in translating or changing statements into equations.

10. Proper amount of drill is given to determine the processes previously learned.

11. The evaluation of group work through tests to determine neatness, accuracy and accomplishment.

12. Test ability to make individual tables and plot the graphs.

Correlation

1. Health - Physical Education
   a. Recording game scores
   b. Recording weight, growth, etc.
2. Social Studies - Language Arts
   a. Production records
   b. Export - Import Records
   c. Natural resources
   d. Spelling
3. Art
   a. Drawings of objects to reveal certain comparisons (Automobile production, Airplane production, Home production)
4. Science - Weather, Rainfall, Conservation

CHAPTER V

RELATIONSHIPS BETWEEN ANGLES AND ARCS OF A CIRCLE

Division of Broad Field: Plane Geometry

Level of Work: Eleventh Grade

Overview - Significance of the Unit

The circle is perhaps the most valuable of all geometric constructions. The wheel is responsible for the spread of our civilization from as far back as the ancient chariot, the covered wagon which was indispensable in the spread of civilization, the stage coach, the steam engine, and the coaches which make up the train.

Now in our modern civilization our means of transportation are entirely dependent upon the circle, which constitutes the very foundation of locomotion. The automobile, airplane, steam ship, motorcycle, bicycle and the diesel engine, have been made possible only as a result of a knowledge of the principles involved in the relationships between angles and arcs of the circle.

The concern in this unit, however, is not so much the use of the circle, but rather the construction and meaning of angles formed with it. The machinist, architect, contractor, designer of mechanical devices, as well as the artistic designer, cannot perform without a knowledge of these principles, and a clear conception of such angles and arcs.

The circular graph also depends fully upon the prin-

ciple of measuring angles.

Suggestions for Introducing and Motivating the Unit

In your previous work in geometry you have had occasion to measure angles. You will recall that the unit of measure has been the degree, although sometimes the right and straight angles were used. You are now going to learn a new method of measuring angles, which is of much importance and value in solving certain problems and exercises. Angles which are formed by two lines which intersect or touch a given circle may be relatively measured by the arc or arcs cut off by the sides of the angles. To say that an angle is measured by its arc means that there are as many degrees of angle in the angle as there are degrees of arc in the arc.

Teacher Objectives

General:

1. To aid pupils in doing neat and accurate work.

2. To point out the importance of carrying a task to completion.

3. To emphasize the recognition of relationships.

4. To develop the ability to systematize and classify given data.

5. To develop the ability to think logically and accurately through a problem.

Specific:

1. To teach pupils how to measure angles indirectly.

2. To develop the ability to exercise spatial imagination.

3. To emphasize the importance of discovering new geometric facts.

4. To emphasize the recognition of angle and arc relationships.

5. To teach the pupil to recognize contributions the circle has made toward the progress of civilization.

6. To develop an appreciation of the beauty of geometric forms in nature, art, and architecture.

7. To develop an understanding of the units of measurement of angles and arcs.

8. To teach pupils to understand important theorems and corollaries pertaining to the circle.

Pupil Objectives

1. To develop a degree of interest in mahtematics which will encourage the pupil to continue in the study.

2. A desire to make precise statements.

3. A desire to estimate in advance the solution of a problem.

4. A desire for thoroughness.

5. Willingness to concentrate on problems.

6. A desire to do neat work.

7. A desire to grow mentally, to improve former records.

8. To discover the use of the circle in industry.

9. To discover angles of the circle in nature.

10. To discover the angles of the circle in art.

11. To discover the angles of the circle in architecture.

12. To observe biological formations of angles in animate and inanimate specimen.

13. To be able to classify all angles in connection with the circle.

Pupil Problems and Needs Anticipated in This Unit

Introductory:

1. To discover the application of the principle involved by the use of tangents to circles and tangent circles.

2. To discover the use of parallel lines intercepting arcs on a circle.

3. To discover where angles within the circle are used in industry, art, and other life situations.

Developmental:

4. To learn how to measure an angle if the vertex falls within the circle.

5. To learn how to measure an angle if the vertex is on the circle.

6. To learn how to measure an angle if the vertex falls outside of the circle.

Culminating:

7. To summarize and interpret or evaluate the relationships between angles and arcs of a circle.

Learning Experiences - Activities

Introductory:

1. Visit factories, mills and machine shops.

2. Observe household trinkets and gadgets of circular nature containing angles.

3. Notice all vehicles for their wheel construction.

4. Look for angles in the circle found in nature, the biological laboratory, animate and inanimate objects.

5. Bring to class an old clock. Take it apart in class and do likewise with any other device that may contain angles or arcs.

6. Observe the use of the pulley in lifting heavy objects and bring it to class if possible.

7. Observe bicycle chains, and their similarity to external tangents when laced around their sprockets.

8. Bring toy Yo-yos to class and compare them with tangents.

Developmental:

9. If AB and CD are diameters of a circle, the arc $\overset{\frown}{AD}$ is equal to the arc $\overset{\frown}{BC}$.

10. Examine a wheel or several wheels in the class collection. Count the number of equal arcs cut off by the spokes.

11. Measure the angles found in the sprockets of a clock and other circular gadgets which have been collected and tell the number of degrees in their arcs.

12. The sides of a triangle inscribed in a circle subtend arcs of 120°, 130° and 110°. How many degrees are there in each angle of the triangle?

13. Draw lines on the face of the old clock brought to class from 2 to 4 and from 6 to 11. Find the number of degrees in each inscribed angle.

14. AB is a tangent and AC a secant intersecting outside a circle and intercepting arcs of 30° and 80°. Find the size of the angle formed at A.

15. ED and EF are two secants forming an angle of 40° at E and intersecting arcs, $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$. The lesser arc $\overset{\frown}{AB}$ is 600. How many degrees are there in arc $\overset{\frown}{CD}$?

Culminating:

1. Have pupils make a list of various terms used in this unit, such as arcs, central angles, angles formed by intersecting on the circle, angles formed by a tangent and

a chord. Angles formed by the intersection of a tangent and a secant, two secants, and two tangents intersecting outside a circle. Point out examples of these cases observed in mills, factories and architectural designs.

17. Assemble all gadgets collected by pupils on a table in the room and have pupils point out the theorem or corollary which applies to each.

18. Make a bulletin board display of original drawings, clippings from magazines and biological drawings.

19. Assemble miniature cars, wagons, airplanes and calculate size of the angles and arcs formed by the wheel spokes.

20. Construct circles containing as many of the various angles, and arcs studied in this unit, and calculate the number of degrees in each.

Materials

1. The pupils' notebooks.
2. A ruler or a combination ruler and protractor.
3. A protractor.
4. The pupils' compasses.
5. Blackboards, erasers and crayon.
6. Blackboard protractors.
7. Blackboard rulers.
8. Blackboard pointers.

9. Blackboard compasses

10. Filing case

11. Pencil sharpeners

12. Pieces of cardboard, sticks and string, to be used in building up models illustrating geometric principles.

13. Several wooden triangles with angles equal to 45°, 30° and 60°.

14. A box of colored crayons.

15. Several pairs of scissors.

16. A collection of pictures to illustrate the use of angles of the circle as used in art, architecture, and engineering.

17. A bulletin board for posting assignments and exhibiting good work.

18. Textbook used in the course: Morgan, Frank M., Foberg, John A., and Breckenridge, W. E., Plane Geometry. Dallas: Houghton Mifflin Company, 1943.

19. Bryce, W. H., Some Noteworthy Properties of the Triangle and Its Circles, Boston: D. C. Heath & Co., 1942.

20. Hendrick, E. R., Constructive Geometry. New York: Macmillan Company, 1916.

21. Schorling, Raleigh, The Teaching of Mathematics. Ann Arbor: The Ann Arbor Press, 1936.

22. Breslich, Ernst R., The Technique of Teaching Secondary School Mathematics. University of Chicago Press, 1930, pp. 126-212.

23. The Mathematics Teacher, Volume 37, No. 6. October, 1944.

24. Burns, Frances M., "The Use of Models in the Teaching of Geometry," Mathematics Teacher, 37: 272-277.

25. Film: "Modes and Motors," one reel, 16mm., sound, 10 min., 1939. Free: General Motors Corp., New York City.

26. Film: "Development of Transportation," one reel, 16 and 35mm., sound, 10 min., 1939. Rent: $1.50 - $2.00, AM, BY. Sale: $45.00 Erpi, Twy. Dayton, Ohio.

Suggested Evaluative Procedures

1. Divide the class into three groups, A, B and C. Group A will summarize the use of angles formed inside the circles in as many fields of life as can be pointed out. For example: machine shops, architecture, art, industry, nature, etc. Group B will summarize the use of angles whose vertices fall on the circle. Group C will summarize the use of angles whose vertices fall outside the circle.

2. Enumerate other subject fields.

3. Have pupils make original problems and give them to the class to be solved.

4. Have pupils make up completion and true-false tests covering all phases of the unit.

5. Give some of these tests to the class and have pupils check them for thoroughness of content.

6. Have pupils make picture notebooks by cutting the

pictures from magazines and showing the use of arcs and angles of the circle. Arrange in a classified manner.

7. Prepare a written report of the development of architecture, pointing out the uses of arcs and angles.

8. Give a history of the development of the airplane.

9. Give a report on a cross-section of a tree naming the various sections and pointing out concentric circles.

10. Give pupils an opportunity to ask questions about any phase of the unit.

11. Prepare and give a well balanced test involving true-false, completion and a sufficient number of well selected problems involving all the various angles and arcs.

12. Evaluate individual students as to their understandings, attitudes, personal, social and moral traits, as well as to skills and objectives accomplished.

Correlation

The various activities extend into the following subject fields:

1. Language Arts
   a. See that the correct form of composition is used in the historical development of the various architectural styles.
2. b. Write biographical sketches of Thales, Euclid

and other mathematicians.

c. Develop skills of reaching, spelling, writing and listening.

d. Make a word list of new terms in the field of mathematics.

e. Write a skit using terms as, Miss Circle, Mr. Chord, Mr. Tangent, Mr. Angle, Miss Arc, and others as characters.

2. Social Studies:

a. Trace the historical development of the wheel from the ox cart to the present means of transportation.

b. Locate the different time belts in the United States.

3. Home and Vocational Arts:

a. Point out the uses of circular designs in household articles such as: floor coverings, table cloths, bed spreads, tea towels, draperies, etc.

4. Biology:

Examine biological specimen of circular nature such as: sea-anemone, sea-urchin, hydra, and the starfish.

5. Physics:

Develop formulas for lifting weights by the use

of pulleys.

6. Aeronautics:

   a. Name the different types of airplanes and give their uses.

   b. Make drawings and models of various kinds of airplanes.

   c. Visit airports and obtain schedules and calculate the time it will take to fly from Dallas to other parts of the world.

CHAPTER VI

SOLID FIGURES COME TO LIFE

Division of Broad Field: Solid Geometry

Level of Work: Eleventh Grade

Overview - Significance of the Unit

It is quite evident in Solid Geometry, as in any other course, that the teacher is confronted with pupils of varying abilities. Quite a few pupils display better preparedness and mental gifts that enable them to readily visualize the various figures encountered in Solid Geometry, while on the other hand, some seem to have only a minimum of imagination. Usually those who lack this power of imagination to a certain extent, which is so important in geometry, possess mechanical ability in varying degrees. To assist this group, we shall endeavor to put into play models of various theorems and solids which will be made by the individual student.

In addition to making figures come to life through models, each pupil will be guided in classifying certain crystalline solids as to basic structure. This will in turn call for instruction in the use of the microscope as a measure of correlation with the other sciences.

It is hoped that through the use of the activities set up in this unit there will result a better understanding of the purpose of mathematics as well as Solid Geometry.

This lack of understanding on the part of the student as to the nature of mathematics is considered by the writer to be one of the greatest handicaps today to a wider acceptance of mathematical study.

Teacher Objectives

General:

1. To develop specific knowledges that are useful in everyday life.

2. To help pupils in appraising such cultural values as beauty of design, originality and the power of applied mathematics.

3. To influence logical reasoning and reflective thinking.

4. To develop thinking in terms of quantitative relations.

5. To develop neatness.

Specific:

1. To develop spatial imagination.

2. To help each pupil realize his need for more of the theories of Solid Geometry.

3. To develop an understanding of perspective.

4. To develop precision in mathematical construction.

5. To show the importance of Solid Geometry to man's welfare in a three dimensional world.

Pupil Objectives

1. To acquire knowledge of the facts and principles of geometry.

2. To design model proofs and develop the memory.

3. To meet requirements for a college major in mathematics.

4. To prepare for the study of science and advanced mathematics.

Pupil Problems and Needs Anticipated in this Unit

Introductory:

1. To become acquainted with Solid Geometry.

2. To develop a sense of perspective.

3. To develop the imagination.

Developmental:

1. To learn about lines and planes in three dimensions.

2. To learn about lines and planes in industry.

3. To build extended views on dihedral and polyhedral angles.

4. To introduce spatical loci.

5. To study and set up formulas for general and special cases of polyhedrons, cylinders, cones and frustrums of cones.

6. To study the sphere.

Culminating:

1. To summarize in order to determine whether each student has gained adequate information.

2. To summarize the unit of work in its entirety and connect various phases in order to get a clear conception of the whole.

Learning Experiences - Activities

Introductory:

1. Compare two-dimensional space with three-dimensional space by discussing fundamental truths in Plane Geometry along with those in Solid Geometry.

2. Give examples of perspective such as: looking down a railroad track, down a line of telephone poles, etc.

3. Look at various solids from different angles and then construct them on paper.

4. Each pupil makes up problems such as the following to draw: A baseball resting on a plane.

5. Field trips will be made to view buildings under construction in order to see relations between the floors and hoists. This can be used as a method of helping the students see parallel and perpendicular planes.

6. Films will be shown.

7. Visits will be made to the movies to see three-dimensional pictures.

Developmental:

1. Show with drawings the definitions of terms.

2. Draw theorem figures.

3. Construct theorem figures with wire, cardboard and tape.

4. Study pictures of buildings and other structures giving truths of lines and planes.

5. Study problems in everyday life in connection with each theorem.

6. Gather information on the elevation of several principle cities of the United States and compare.

7. Study and use the surveyor's transit.

8. Formulate exercises on leveling in house building.

9. Have a contractor to speak on the importance of geometry in building.

10. Models of clay, cardboard, wood and plastic will be made of special and some general solids.

11. Discussions will be conducted in which students will give examples of solids about us.

12. Students will make microscopic study of crystalline solids and classify each generally.

13. Each student will make a study in astronomy in connection with the celestial sphere.

14. Through the use of the atlas, students will find

the latitude and longitude of various significant places.

15. Present a meteorologist to speak on the importance of geometry in weather forecasting.

Culminating:

1. Plan a project such as a "Sodid Tree," a "Polyhedron Plant," etc.

2. Present an assembly program on which students will give the importance of Solid Geometry in everyday living.

3. Display of exhibits, inviting students, teachers, principal and parents.

4. Have Servicemen to talk on geometry in their specific branches of service.

Materials

1. Breslich, Ernst R., The Techniques of Teaching Secondary School Mathematics. Chicago: University of Chicago Press, 1930.

2. Dallas Independent School District, High School Mathematics (Outline for Teachers), 1951.

3. Schorling, Raleigh, The Teaching of Mathematics. Ann Arbor: The Ann Arbor Press, 1936.

4. Baker, Robert H., Introduction to Astronomy. New York: D. Van Nostrand Co., Inc., 1947.

5. Grandal, Florence A., The Romance of Astromony. New York: Macmillian Company, 1943.

6. Betz, William, Basic Mathematics. Ginn and Company, 1942.

7. Jeans, James, The Stars in Their Courses, Macmillian Company, 1943.

8. Johnson, James, Applied Mathematics. Milwaukee: Bruce Publishing Company, 1939.

9. Morgan, Breckenridge. Solid Geometry, Boston: Houghton-Mifflin Company, 1946.

10. Schorling and Clark, Mathematics in Life. New York: World Book Company, 1939.

11. Morgan, Foberg and Breckenridge, Plane Geometry. Boston: Houghton, Mifflin Company, 1943.

12. Sommers, Hobart H., Living Mathematics Reviewed. Chicago: Wilcox and Follett Co., 1943.

13. The World Book Encyclopedia, Vol. 7. Chicago: Field Enterprises, Inc., 1951.

14. The World Atlas.

15. Hogben, Lancelot, Mathematics for the Million. New York: Horton & Co., Inc., 1943.

The Mathematics Teacher

1. Brown, Kenneth, "Why Teach Geometry", March, 1950. pp. 103-107.

2. Jones, Phillip, "Early American Geometry," January, 1944. p. 3.

3. Joseph, Margaret, "Accurate Drawings of Three Dimensional Figures," December, 1944, p. 351.

4. Larson, Clara, "The Art of Teaching," April, 1942. pp. 182-184.

5. Peak, Phillip, "Continuity in Geometry," December, 1944. pp. 360-363.

6. Sitomer, Harry, "The Place of Experimentation in Plane Geometry," March, 1944. p. 122.

7. Spearman, Ethel, "Looking at Solid Geometry Through Perspective," April, 1941. p. 147.

Film:

"The Locus," Visual Aids Library, Dallas Independent

School District.

"Lines and Angles," Visual Aids Library, Dallas Independent School District.

Other Materials: Scissors, paste, straight edges, compasses, cardboard, electrical wiring, crystalline solids, microscope, surveyor's transit, paper, colored pencils, clay, wood and plastic.

Suggested Evaluative Procedures:

Teacher observation for contributions of students to class discussions and practical applications made. Have students make up practical problems on various units. Work exercises and construct figures on blackboard. Have individual discussions. Have paper and pencil quizzes.

Correlation:

There can be found a definite correlation between Solid Geometry and Art. Perspective, which plays an important part in Solid Geometry, is one of the most vital elements in Art. It also has a definite relationship to the activity of viewing crystalline solids with the use of the microscope. This is used as a medium of correlation between geometry and the sciences.

The methods of various ancient mathematicians and the contributions of certain cultures to mathematics, will be discussed. This is a correlation with the Social Studies.

CHAPTER VII

A TIME SAVING DEVICE - LOGARITHMS

Division of Broad Field: Trigonometry

Level of Work: Twelfth Grade

Overview - Significance of the Unit

With very big numbers the discrepancy between the amount of labor involved in multiplication (or division) as compared with addition (or subtraction) becomes greater and greater. So it is a very large economy of effort if we can reduce all multiplication to the addition of two numbers. This is what the invention of logarithms has done for us.

It is the purpose of this unit to set up an organized method of presenting logarithms to high school students. So often facility in the functional use of logarithms is never accomplished because pupils are not taught the actual value of them. Pupils should be taught to realize that logarithms are used as a time-saving device, and consequently, have no advantage over arithmetical methods unless speed and accuracy in the use of the tables, interpolation, and the signs of characteristics can be developed.

Usually in the Third Course Algebra a brief introduction to logarithms is afforded. But, a more thorough unit is presented in connection with Trigonometry because the practical problems here will call for a more diversified use of logarithms.

It is a consensus of opinion that the method of presenting logarithms in parts usually leads to manipulation without understanding. Therefore, to prevent this weakness, through this unit the author is presenting an "over-all" picture to be maintained throughout the whole teaching process.

Suggestions for Introducing and Motivating the Unit

Basic to the study of logarithms is a thorough knowledge of exponents. It should include the:

1. Meaning of exponents.

2. Meaning of zero, negative, and fractional exponents.

3. Four fundamental operations affecting exponents - multiplication, division, raising to a power, and extracting roots.

4. Establishment of the laws of exponents resulting from these four operations.

5. Practice with mixed groups of exercises including types such as these:

$(x^3)^4 =$ $\qquad$ $10^3 \cdot 10^2 =$

$(x^6)^3 =$ $\qquad$ $(10^{6+2})^3 =$

$(x^{-3})^2 =$ $\qquad$ $(10^{-6}) \div (10^{-2}) =$

$x^{\frac{1}{2}} \cdot x^{\frac{1}{3}} =$ $\qquad$ $10^3 \div 10^{\frac{1}{2}} =$

$x^{\frac{1}{2}} \div x^{\frac{1}{3}} =$ $\qquad$ $\sqrt{x^6} =$

$x^6 \div x^2 =$

$\sqrt[3]{x^6} =$

$(10^4)^3 =$

$x^3 \cdot x^{2\frac{1}{2}} =$

$10^{6.42} \div 10^{2.41} =$

$10^{\frac{1}{2}} \cdot 10^{\frac{2}{3}} =$

$10^{2.443} \cdot 10^{6.844} =$

$10^{\frac{1}{2}} \div 10^{\frac{2}{3}} =$

$$\frac{(x^3)^4 \cdot (x^2)^3 \cdot (x^3)^3 \cdot (x^{-2})^3}{(x^2)^4 \cdot (x^{\frac{1}{2}})^6 \cdot x^0} =$$

$$\frac{(10^3)^2 \cdot (10^2)^3 \cdot (10^4)^3}{10^0 \cdot (10^3)^5} =$$

$$\frac{(10^{3.23})^3 \cdot (10^{6.44})^2 \cdot 10^4}{(10^{2.21})^3 \cdot (10^{3.42})^2} =$$

$$\sqrt{\frac{(10^3)^4 \cdot (10^2)^2 \cdot (10^3)^2}{(10^4)^3 \cdot 10^1}} =$$

$$\sqrt[3]{\frac{(10^{3.6442})^3 \cdot (10^{6.2442})^2}{(10^{3.3311})^2}} =$$

Many exercises are long and tedious, when solved by the common processes of elementary arithmetic. This is verified by working out in detail and by keeping a record of the time consumed in this exercise.

$$\sqrt{\frac{(492)^3 \cdot (492)^2 \cdot (492)}{(4920)^2}}$$

Observe the number of opportunities for making errors.

Teacher Objectives

General:

1. To develop accuracy and facility in the fundamental processes.

2. To develop the knowledge and power to apply mathematical concepts.

3. To develop a realization of the specific knowledge useful in life.

4. To develop an appreciation of disciplinary values.

5. To develop a concept of time.

6. To develop an appreciation of cultural values.

Specific:

1. To develop the ability to pronounce, spell, and use in appropriate context such words and terms as: characteristic, mantissa, logarithm, reciprocal, anti-logarithm, base, power, exponent, root, exponential-form equation, logarithmic-form equation, interpolation, mantissa table, and the like.

2. To develop the ability to distinguish the characteristic and the mantissa of a given logarithm.

3. To develop the ability to find the characteristic of the logarithm of any number greater than 1 by counting the integral places in the number.

4. To develop the ability to find the characteris-

tic of the logarithm of any number less than 1 by counting the number of zeros immediately to the right of the decimal point.

5. To show how to find the logarithm of 1.

6. To show how to find the appropriate mantissa for a given number by consulting the mantissa table.

7. To develop facility in finding the anti-logarithm of a given mantissa by consulting the mantissa table.

8. To show how to interpolate when finding mantissas of numbers and when finding anti-logarithms of mantissas.

9. To develop the ability to add logarithms when multiplying anti-logarithms; to subtract logarithms when dividing anti-logarithms.

10. To develop the ability to divide logarithms by the appropriate number when extracting roots; to multiply when raising to powers.

11. To show how to make use of -10 or some other negative quantity in finding the characteristics of logarithms of numbers less than 1.

12. To develop the ability to locate the decimal point in anti-logarithms by considering the characteristics.

13. To develop the ability to find the anti-logarithms of logarithms from whose characteristics 10, or multiples of 10, or other quantities are to be subtracted.

Pupil Objectives

1. To be able to attack problems of interest to them.

2. To be able to analyze their problems.

3. To be able to formulate possible solutions.

4. To be able to gather data.

5. To be able to simplify long mathematical processes of multiplication and division.

6. To be able to interpret findings and apply them to useful purposes.

7. To be able to meet the needs of better and faster means of computation.

8. To be able to devise other shortcuts in performing the fundamental operations.

Pupil Problems and Needs Anticipated in this Unit

1. The importance of logarithms

2. Symbolism

3. The relation of a system of logarithms to its base (especially the relation of the commonly taught system to 10).

4. The relation of the exponential form of expressing such equations.

5. The meaning of exponent as related to the meaning of power.

6. The relation of multiplication, division, rais-

ing to powers, and extracting roots by means of logarithms to the laws of exponents.

7. How mantissa tables are constructed.

8. The processes involved in multiplying, dividing, raising to powers and extracting roots by means of logarithms.

9. Why characteristics change and mantissas remain the same when anti-logarithms are multiplied, or divided by 10.

10. How logarithms of numbers less than 1 may be written in such form as to make use of -10.

11. The relation of the slide rule to logarithms.

12. The processes involved in making tabular interpolations.

13. The advantage of using a stereotyped form of equation when working with logarithms.

14. The work of Napier and Briggs, each of whom contributed much to the invention of logarithms.

15. The power of mathematics in that it includes such economical means of getting work done as logarithms.

16. The part played by logarithms and related mathematical processes and devices in developing such sciences as navigation, astronomy, engineering, and economics.

17. Algebra as a branch of mathematics which furnishes the essential framework for the work of logarithms.

Learning Experiences - Activities

1. The vital problem is to learn how to write numbers to the same base.

2. Understand and expand the table of powers of 10, beginning with:

$$10^0 = 1$$
$$10^1 = 10$$

etc.

3. Observe how the numbers in the original exercise may be placed in the table of tens.

$10^0 = 1$		$10^{1.+} = 49.2$		$10^3 = 1{,}000$	
$10^{0.+} = 4.92$		$10^2 = 100$		$10^{3.+} = 4{,}290$	
$10^1 = 10$		$10^{2.+} = 492$		etc.	

4. Make a skeleton of the original exercise, verifying from the above table, as:

$$\sqrt{\frac{(10^{2.\text{——}})^3 \cdot (10^{1.\text{——}})^2 \cdot (10^{0.\text{——}})}{(10^{1.\text{——}})^2}}$$

5. Since the numbers lie between the powers of ten, then, when they are written to the base of 10, their exponents must be between the powers above and below them. A number must be the power above it in the table, plus some fraction. This fraction, called the mantissa is written in decimal form and can be found in a table of mantissas.

6. Using the table completes the process of writing the given numbers to the base of 10. The exercise now be-

comes:

$$\sqrt{\frac{(10^{2.6920})^3 \cdot (10^{1.6920})^2 \cdot (10^{0.6920})}{(10^{3.6920})^2}}$$

It is evident that:

a. The characteristic, or number to the left of the decimal point in the exponent, varies according to the size of the number or its position in the table of powers of 10.

b. The mantissa is the same when the order of the numerals is the same. If the order changes, the mantissa changes.

7. The operations in the original exercise are listed. After changing to the same base the replacement operations are noted as:

raising to a power - multiplication of exponent by the power

multiplication - addition of exponents

division - subtraction of exponent of divisor from exponent of dividend

extracting a root - dividing the exponent of the radicand by the index of the root

8. Performing the operations, step by step, presents the following aspects

$$\sqrt{\frac{(10^{8.0760}) \cdot (10^{3.3840}) \cdot (10^{0.6920})}{(10^{7.3840})}}$$

$$\sqrt{\frac{10^{12.1520}}{10^{7.3840}}}$$

$\sqrt{10^{4.7680}}$

$10^{2.3840}$

9. Know that the exponent of the power to which 10 must be raised to equal the given number is the logarithm of the number.

10. Understand that when an exercise such as the one above, is reduced to a single expression - 10 to some power- one more step becomes necessary. It must be changed back to a number that will be the answer to the exercise.

11. The number corresponding to a given logarithm is called the anti-logarithm. This necessitates using a logarithm table in the reverse order from finding the logarithm. Looking up the mantissa inside the table and going out to find the number gives 242. From the table of powers of 10 a characteristic of two locates the number between 100 and 1,000. All numbers between these two locations consist of three places. Hence, the location of the decimal point after the number is thus: 242.

The above steps serve to give the "over-all" picture of logarithms and to help the student understand what he is trying to do. However, a number of further details must be undertaken in completing the study of logarithms. They include:

1. The logarithm of fractions, written in decimal form.

2. Interpolating for logarithms and anti-logarithms.

3. How to express the mantissa of any number in a form that will be positive.

4. The generalization of the laws for characteristics.

5. The meaning and use of cologarithms.

6. The writing and solution of exercises in the brief logarithmic form.

7. Historical material on John Napier and Henry Briggs.

8. A wide variety of practical exercises, including the compound interest formulas.

9. The exponential equations.

10. The slide rule - a concrete use of logarithms.

11. The reworking of the original exercise, in a complete manner, by logarithms, and a comparison of the time required by the arithmetical and by the logarithmic processes.

Materials.

1. Brink, Raymond W., Plane Trigonometry. New York: D. Appleton-Century Company, 1940.

2. Brown, Claude H., The Teaching of Secondary Mathematics. New York: Harper and Brothers

3. Kinney, Lucien and Purdy, C. Richard, Teaching Mathematics in the Secondary Schools. Pinehard and Company, Inc, 1940.

4. Wentworth, George and Smith, David Eugene, Plane Trigonometry. Boston: Ginn and Company, 1943.

5. American Mathematical Monthly, 49:671, December, 1942: 50:55, January, 1943.

6. Educational Research Bulletin, 19:479, November, 1940.

7. Journal of English Education, 38:393, January, 1948.

8. School Science and Mathematics, 39:763; 40:161.

9. Science News Letter, 61: 296, May 10, 1952.

10. Slide rules

11. Logarithmic tables from the textbook.

Evaluative Procedures

1. Assign oral and written reports on the history of logarithms.

2. Class discussions on the history of logarithms.

3. Pupil appraisal and expressions on usefulness of logarithms.

4. Pupil discussions of problems in other courses where logarithms may be used.

5. Tests on each phase of unit to assure understanding of fundamentals.

6. Individual board demonstrations by pupils of specific problems.

In the final appraisal of this unit, an overall test will be given to determine whether the student has mastered the basic skills and developed proper attitudes for further applications.

Correlation

In solving triangles, and in many other problems, the work of numerical computation can be greatly lessened by the use of logarithms. Logarithms enable us to replace the processes of multiplication and division by the simpler ones of addition and subtraction and to replace the taking of powers and roots by multiplication and division. In view of this the utility of logarithms, particularly in the sciences, can easily be realized. They are indispensable in drawn out computations wherein other instruments such as the slide rule and numerical calculator are not available.

CHAPTER VIII

CONCLUSION

The planning of such resource units as included in this thesis requires careful and detailed analysis of the material; the formulation of the general and specific objectives; the selection and arrangement of topics and activities; and provision for further expansion of the unit. The actual preparation of the plan of each unit is important in that it forces the teacher to compare the different topics and details within the unit with reference to their relative importance, thus giving a basis for wise selection and appropriate emphasis of the subject matter to be included within the unit. It also compels the teacher to take into consideration the relative difficulty of the various parts of the subject matter and, in this way, facilitates the preparation of differentiated assignments in adjusting the requirements of the course to different levels of ability among the students.

With the work definitely planned and organized, the developmental work is made more meaningful than it would otherwise be and adds understanding, interest, and motive to the activities of the students during this period of study.

As has been previously mentioned, the resource unit is flexible in content which means that the units included in this thesis are by no means complete, but, through fur-

ther enriched experiences much material can be gathered and added to them.

The writer hopes that as many teachers of secondary mathematics can be exposed to these units as possible, and, through an evaluation of them may receive some aid in the presentation of the particular topics developed and also, receive ideas for further developments and additions along this line of teaching aids.

BIBLIOGRAPHY

(Due to the fact that within each unit there is a list of sources under the heading, "Materials", only those sources are listed here which were used as general references.)

Books.

1. Butler, Charles H. and Wren, F. Lynwood, The Teaching of Secondary Mathematics. New York: McGraw-Hill Company, Inc., 1951.

2. Currier, C. H., A Course in General Mathematics. New York, Macmillan and Company, 1929.

3. Evans, George, The Teaching of High School Mathematics. Boston: Houghton-Mifflin, 1911.

4. Hassler, J. O., and Smith, R. R., The Teaching of Secondary Mathematics. New York, Macmillan Company, 1930.

5. Lee, J. Murray and Lee, Dorris May, The Child and His Curriculum. New York: Appleton-Century-Crofts, Inc., 1950.

6. Mehl, Marie A., and Mills, Hubert H., Teaching in Elementary Schools. New York; The Ronald Press Company, 1950.

7. Nunn, T. Percy, The Teaching of Algebra. New York: Longmans, Green & Company, 1919.

8. Schorling, Raleigh, The Teaching of Mathematics. Ann Arbor: Ann Arbor Press, 1936.

9. Schultze, Arthur, The Teaching of Mathematics in Secondary Schools. New York: Macmillan Company, 1912.

10. Schuster, C. N., "The Use of Measuring Instruments in Teaching Mathematics," Third Yearbook of the National Council of Teachers of Mathematics. New York: Bureau of Publications, Teachers College, Columbia University, 1928.

11. Schuster, C. N., and Fred L. Bedford, Field Work in Mathematics. New York: American Book Company, 1935.

12. Touton, Frank, The Roles of Purpose in the Teaching of Secondary Mathematics. Boston: Ginn and Company, 1932.

13. Young, J. W. A., The Teaching of Mathematics. New York: Longmans Green and Company, 1924.

Periodicals

1. Henderson, K. B., "Use of Resource Units," School Science and Mathematics, 49:345-9, May, 1949.

2. Miller, G. A., "Useful Mathematics," School and Society, 31:646-7, May 10, 1930.

Other Material

1. Committee on Definitions and Principles, Dallas Independent School District, Manual for Using the Form for Planning and Developing a Resource Unit, 1951.